Problems on Permutations and Combinations - Solved Examples

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(Set 1)

1. Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

A. 24400  
B. 21300  
C. 210  
D. 25200

\[
\text{Number of ways of selecting 3 consonants from 7} = \binom{7}{3} \\
\text{Number of ways of selecting 2 vowels from 4} = \binom{4}{2} \\
\text{Number of ways of selecting 3 consonants from 7 and 2 vowels from 4} = \binom{7}{3} \times \binom{4}{2} \\
= (7 \times 6 \times 5 \times 3 \times 2 \times 1) \times (4 \times 3 \times 2 \times 1) = 210 \\
\]

It means we can have 210 groups where each group contains total 5 letters (3 consonants and 2 vowels).

\[
\text{Number of ways of arranging 5 letters among themselves} = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120 \\
\]

\[
\text{Hence, required number of ways} = 210 \times 120 = 25200 \\
\]

answer with explanation

Answer: Option D

Explanation:

Number of ways of selecting 3 consonants from 7  
= \( \binom{7}{3} \)  
Number of ways of selecting 2 vowels from 4  
= \( \binom{4}{2} \)  
Number of ways of selecting 3 consonants from 7 and 2 vowels from 4  
= \( \binom{7}{3} \times \binom{4}{2} \)  
= \( \frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \frac{4 \times 3}{2 \times 1} = 210 \)  

It means we can have 210 groups where each group contains total 5 letters (3 consonants and 2 vowels).

Number of ways of arranging 5 letters among themselves  
= 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120 

Hence, required number of ways  
= 210 \times 120 = 25200
2. In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?

A. 159  
B. 209  
C. 201  
D. 212

**Answer with explanation**

**Answer:** Option B

**Explanation:**

In a group of 6 boys and 4 girls, four children are to be selected such that at least one boy should be there.

Hence we have 4 options as given below

- We can select 4 boys ...(option 1)
  Number of ways to this = \(^6C_4\)

- We can select 3 boys and 1 girl ...(option 2)
  Number of ways to this = \(^6C_3 \times \(_4C_1\)

- We can select 2 boys and 2 girls ...(option 3)
  Number of ways to this = \(^6C_2 \times \(_4C_2\)

- We can select 1 boy and 3 girls ...(option 4)
  Number of ways to this = \(^6C_1 \times \(_4C_3\)

Total number of ways

\[= \(^6C_4 + \(_6C_3 \times \(_4C_1 + \(_6C_2 \times \(_4C_2 + \(_6C_1 \times \(_4C_3\)
\]

\[= \(_6C_2 + \(_6C_3 \times \(_4C_1 + \(_6C_2 \times \(_4C_2 + \(_6C_1 \times \(_4C_3\)
\]

\[= 6 \times 5 \times \frac{43}{2 \times 1} + 6 \times 5 \times \frac{43}{2 \times 1} \times 4 + 6 \times 5 \times \frac{43}{2 \times 1} \times 32 \times 1 \times 6 \times 4 + 6 \times 5 \times 2 \times 1 \times 4 \times 32 \times 1 + 6 \times 4 + 6 \times 5
\]

\[= 15 + 80 + 90 + 24 = 209 = 15 + 80 + 90 + 24 = 209\]

3. From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there in the committee. In how many ways can it be done?

A. 624  
B. 702
Answer with explanation

**Answer:** Option C

**Explanation:**

From a group of 7 men and 6 women, five persons are to be selected with at least 3 men.

Hence we have the following 3 options.

We can select 5 men ...(option 1)
Number of ways to do this = \(7C_5\)

We can select 4 men and 1 woman ...(option 2)
Number of ways to do this = \(7C_4 \times 6C_1\)

We can select 3 men and 2 women ...(option 3)
Number of ways to do this = \(7C_3 \times 6C_2\)

Total number of ways
= \(7C_5 + (7C_4 \times 6C_1) + (7C_3 \times 6C_2)\)
= \(7C_2 + (7C_3 \times 6C_1) + (7C_3 \times 6C_2)\) \[\because nC_r = nC_{n-r}\]

= \(7\times 62\times 1+7\times 6\times 53\times 2\times 1\times 6=7\times 62\times 1+7\times 6\times 53\times 2\times 1\times 6+7\times 6\times 53\times 2\times 1\times 6\times 52\times 1+7\times 6\times 53\times 2\times 1\times 6\times 52\times 1\)

=21+210+525=756=21+210+525=756

4. In how many different ways can the letters of the word 'OPTICAL' be arranged so that the vowels always come together?

A. 610
B. 720
C. 825
D. 920

Answer with explanation

**Answer:** Option B
Explanation:

The word 'OPTICAL' has 7 letters. It has the vowels 'O','I','A' in it and these 3 vowels should always come together. Hence these three vowels can be grouped and considered as a single letter. That is, PTCL(OIA).

Hence we can assume total letters as 5 and all these letters are different.
Number of ways to arrange these letters
=\(5! = 5 \times 4 \times 3 \times 2 \times 1 = 120\)

All the 3 vowels (OIA) are different
Number of ways to arrange these vowels among themselves
=\(3! = 3 \times 2 \times 1 = 6\)

Hence, required number of ways
=\(120 \times 6 = 720\)

5. In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together?

A. 47200  
B. 48000  
C. 42000  
D. 50400

Answer with explanation

Answer: Option D

Explanation:

The word 'CORPORATION' has 11 letters. It has the vowels 'O','O','A','I','O' in it and these 5 vowels should always come together. Hence these 5 vowels can be grouped and considered as a single letter. That is, CRPRTN(OOAIO).

Hence we can assume total letters as 7. But in these 7 letters, 'R' occurs 2 times and rest of the letters are different.

Number of ways to arrange these letters
=\(7!2! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 2 \times 1 = 2520\)

In the 5 vowels (OOAIO), 'O' occurs 3 and rest of the vowels are different.

Number of ways to arrange these vowels among
themselves \(=5!3!=5\times4\times3\times2\times1=20\)

\(=5!3!=5\times4\times3\times2\times1=20\)

Hence, required number of ways
\(=2520\times20=50400\)

6. In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?
A. 1  
B. 126  
C. 63  
D. 64

answer with explanation

Answer: Option C

Explanation:

We need to select 5 men from 7 men and 2 women from 3 women.

Number of ways to do this
\(\binom{7}{5} \times \binom{3}{2}\)
\(=7C_5 \times 3C_2\)
\(=7C_2 \times 3C_1 \quad [\because \ \binom{n}{r} = \binom{n}{(n-r)}]\)
\(=\frac{7\times6}{2\times1}\times\frac{3\times2}{1}\times1\times3=21\times3=63\)

7. In how many different ways can the letters of the word 'MATHEMATICS' be arranged such that the vowels must always come together?
A. 9800  
B. 100020  
C. 120960  
D. 140020

answer with explanation

Answer: Option C

Explanation:

The word 'MATHEMATICS' has 11 letters. It has the vowels 'A', 'E', 'A', 'I' in it and these 4 vowels must always come together. Hence these 4 vowels can
be grouped and considered as a single letter. That is, MTHMTCS(AEAI).

Hence we can assume total letters as 8. But in these 8 letters, 'M' occurs 2 times, 'T' occurs 2 times but rest of the letters are different.

Hence, number of ways to arrange these letters
\[= 8! \div (2!) \times (2!)
= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \div (2 \times 1) \times (2 \times 1)
= 10080\]

In the 4 vowels (AEAI), 'A' occurs 2 times and rest of the vowels are different.

Number of ways to arrange these vowels among themselves
\[= 4! \div 2!
= 4 \times 3 \times 2 \times 1 \div 2 \times 1
= 12\]

Hence, required number of ways
\[= 10080 \times 12 = 120960\]

8. There are 8 men and 10 women and you need to form a committee of 5 men and 6 women. In how many ways can the committee be formed?
A. 10420
B. 11
C. 11760
D. None of these

Answer: Option C

Explanation:
We need to select 5 men from 8 men and 6 women from 10 women

Number of ways to do this
\[= \binom{8}{5} \times \binom{10}{6}
= \binom{8}{3} \times \binom{10}{4} \quad [\because \binom{n}{r} = \binom{n}{n-r}]\]
\[= (8 \times 7 \times 6 \times 3 \times 2 \times 1)(10 \times 9 \times 8 \times 7 \times 4 \times 3 \times 2 \times 1)
= 56 \times 210 = 11760\]

9. How many 3-letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed?
The word 'LOGARITHMS' has 10 different letters.

Hence, the number of 3-letter words (with or without meaning) formed by using these letters
\( = \binom{10}{3} \)
\( = 10 \times 9 \times 8 = 720 \)

10. In how many different ways can the letters of the word 'LEADING' be arranged such that the vowels should always come together?
A. None of these  
B. 720  
C. 420  
D. 122

**Answer with Explanation**

**Answer:** Option B

**Explanation:**
The word 'LEADING' has 7 letters. It has the vowels 'E', 'A', 'I' in it and these 3 vowels should always come together. Hence these 3 vowels can be grouped and considered as a single letter. That is, LDNG(EAI).

Hence we can assume total letters as 5 and all these letters are different.
Number of ways to arrange these letters
\( = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120 \)

In the 3 vowels (EAI), all the vowels are different. Number of ways to arrange these vowels among themselves
\( = 3! = 3 \times 2 \times 1 = 6 \)

Hence, required number of ways
= 120 \times 6 = 720

11. A coin is tossed 3 times. Find out the number of possible outcomes.
A. None of these
B. 8
C. 2
D. 1

**answer with explanation**
**Answer: Option B**

**Explanation:**

When a coin is tossed once, there are two possible outcomes: Head(H) and Tale(T)

Hence, when a coin is tossed 3 times, the number of possible outcomes
= 2 \times 2 \times 2 = 8

(The possible outcomes are HHH, HHT, HTH, HTT, THH, THT, TTH, TTT)

12. In how many different ways can the letters of the word 'DETAIL' be arranged such that the vowels must occupy only the odd positions?
A. None of these
B. 64
C. 120
D. 36

**answer with explanation**
**Answer: Option D**

**Explanation:**

The word 'DETAIL' has 6 letters which has 3 vowels (EAI) and 3 consonants(DTL)
The 3 vowels (EAI) must occupy only the odd positions. Let’s mark the positions as (1) (2) (3) (4) (5) (6). Now, the 3 vowels should only occupy the 3 positions marked as (1), (3) and (5) in any order.

Hence, number of ways to arrange these vowels
= \(3! = 3 \times 2 \times 1 = 6\)

Now we have 3 consonants (DTL) which can be arranged in the remaining 3 positions in any order. Hence, number of ways to arrange these consonants
= \(3! = 3 \times 2 \times 1 = 6\)

Total number of ways
= number of ways to arrange the vowels × number of ways to arrange the consonants
= \(6 \times 6 = 36\)

13. A bag contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the bag, if at least one black ball is to be included in the draw?

A. 64  
B. 128  
C. 32  
D. None of these

**Answer with explanation**

**Answer:** Option A

**Explanation:**

From 2 white balls, 3 black balls and 4 red balls, 3 balls are to be selected such that at least one black ball should be there.

Hence we have 3 choices as given below

- We can select 3 black balls...(option 1)
- We can select 2 black balls and 1 non-black ball ...(option 2)
- We can select 1 black ball and 2 non-black balls ...(option 3)

Number of ways to select 3 black balls
= \(3C_3\)

Number of ways to select 2 black balls and 1 non-black ball
= \(3C_2 \times 6C_1\)
Number of ways to select 1 black ball and 2 non-black balls
= \(3C1 \times 6C2\)

Total number of ways
= \(3C3 + 3C2 \times 6C1 + 3C1 \times 6C2\)
= \(3C3 + 3C1 \times 6C1 + 3C1 \times 6C2\) \[∵ nC_r = nC_{(n-r)}\]
= \(1 + 3 \times 6 + 3 \times 6 \times 52 \times 1 = 1 + 18 + 45 = 64\)

14. In how many different ways can the letters of the word 'JUDGE' be arranged such that the vowels always come together?
A. None of these  
B. 48  
C. 32  
D. 64

**Answer with explanation**

**Answer:** Option B

**Explanation:**

The word 'JUDGE' has 5 letters. It has 2 vowels (UE) and these 2 vowels should always come together. Hence these 2 vowels can be grouped and considered as a single letter. That is, JDG(UE).

Hence we can assume total letters as 4 and all these letters are different.

Number of ways to arrange these letters
= \(4! = 4 \times 3 \times 2 \times 1 = 24\)

In the 2 vowels (UE), all the vowels are different. Number of ways to arrange these vowels among themselves
= \(2! = 2 \times 1 = 2\)

Total number of ways = \(24 \times 2 = 48\)

15. In how many ways can the letters of the word 'LEADER' be arranged?
A. None of these  
B. 120  
C. 360  
D. 720
**Answer with explanation**

**Answer:** Option C

**Explanation:**

The word 'LEADER' has 6 letters.

But in these 6 letters, 'E' occurs 2 times and rest of the letters are different.

Hence, number of ways to arrange these letters

\[ = \frac{6!}{2!} = \frac{6 	imes 5 	imes 4 	imes 3 	imes 2 	imes 1}{2 	imes 1} = 360 \]

16. How many words can be formed by using all letters of the word 'BIHAR'?  
A. 720  
B. 24  
C. 120  
D. 60

**Answer with explanation**

**Answer:** Option C

**Explanation:**

The word 'BIHAR' has 5 letters and all these 5 letters are different.

Total number of words that can be formed by using all these 5 letters

\[ = \frac{5!}{5!} = 5! = 5! \]

\[ = 5 	imes 4 	imes 3 	imes 2 	imes 1 = 120 \]

17. How many arrangements can be made out of the letters of the word 'ENGINEERING'?  
A. 924000  
B. 277200  
C. None of these  
D. 182000
Answer with explanation

Answer: Option B

Explanation:

The word 'ENGINEERING' has 11 letters.

But in these 11 letters, 'E' occurs 3 times, 'N' occurs 3 times, 'G' occurs 2 times, 'I' occurs 2 times and rest of the letters are different.

Hence, number of ways to arrange these letters

\[=\frac{11!}{(3!)(3!)(2!)(2!)}=\frac{11\times10\times9\times8\times7\times6\times5\times4\times3\times2(3\times2)(3\times2)(2)(2)}{3\times2}\times3\times2\times2\times2=277200\]

18. How many 3 digit numbers can be formed from the digits 2, 3, 5, 6, 7 and 9 which are divisible by 5 and none of the digits is repeated?
A. 20
B. 16
C. 8
D. 24

Answer with explanation

Answer: Option A

Explanation:

A number is divisible by 5 if the its last digit is 0 or 5

We need to find out how many 3 digit numbers can be formed from the 6 digits (2,3,5,6,7,9) which are divisible by 5.

Since the 3 digit number should be divisible by 5, we should take the digit 5 from the 6 digits (2,3,5,6,7,9) and fix it at the unit place. There is only 1 way of doing this.

Since the number 5 is placed at unit place, we have now five digits (2,3,6,7,9) remaining. Any of these 5 digits can be placed at tens place
Since the digit 5 is placed at unit place and another one digit is placed at
tens place, we have now four digits remaining. Any of these 4 digits can be
placed at hundreds place.

Required Number of three digit numbers
=4×5×1=20=4×5×1=20

19. How many words with or without meaning, can be formed by using all
the letters of the word, 'DELHI' using each letter exactly once?
A. 720  B. 24  C. None of these  D. 120

Answer with explanation
Answer: Option D

Explanation:
The word 'DELHI' has 5 letters and all these letters are different.

Total number of words (with or without meaning) that can be formed using
all these 5 letters using each letter exactly once
= Number of arrangements of 5 letters taken all at a time
= 5P_5 =5! =5×4×3×2×1=120=5! =5×4×3×2×1=120

20. What is the value of ^{100}P_2?
A. 9801  B. 12000  C. 5600  D. 9900

Answer with explanation
Answer: Option D
Explanation:

\[ _{100}P_2 = 100 \times 99 = 9900 \]

21. In how many different ways can the letters of the word 'RUMOUR' be arranged?

A. None of these  
B. 128  
C. 360  
D. 180

**Answer with explanation**

**Answer:** Option D

**Explanation:**

The word 'RUMOUR' has 6 letters.

In these 6 letters, 'R' occurs 2 times, 'U' occurs 2 times and rest of the letters are different.

Hence, number of ways to arrange these letters

\[ = 6!(2!)^2 = 6 \times 5 \times 4 \times 3 \times 2 \times 2 = 180 \]

22. There are 6 periods in each working day of a school. In how many ways can one organize 5 subjects such that each subject is allowed at least one period?

A. 3200  
B. None of these  
C. 1800  
D. 3600

**Answer with explanation**

**Answer:** Option C

**Explanation:**

**Solution 1**
5 subjects can be arranged in 6 periods in $6P_5$ ways.

Any of the 5 subjects can be organized in the remaining period ($5C_1$ ways).

Two subjects are alike in each of the arrangement. So we need to divide by $2!$ to avoid overcounting.

Total number of arrangements

$$= 6P_5 \times 5C_1 \times 2! = 6 \times 5 \times 2! = 1800$$

**Solution 2**

5 subjects can be selected in $5C_5$ ways.

1 subject can be selected in $5C_1$ ways.

These 6 subjects can be arranged themselves in $6!$ ways.

Since two subjects are same, we need to divide by $2!$

Therefore, total number of arrangements

$$= 5C_5 \times 5C_1 \times 6! \div 2! = 5 \times 1 \times 6! \div 2! = 1800$$

**Solution 3**

Select any 5 periods ($6C_5$ ways).

Allocate a different subject to each of these 5 periods (1 way).

These 5 subjects can be arranged themselves in $5!$ ways.

Select the 6th period (1 way).

Allocate a subject to this period ($5C_1$ ways).

Two subjects are alike in each of the arrangement. So we need to divide by $2!$ to avoid overcounting.

Therefore, required number of ways

$$= 6C_5 \times 5C_1 \times 5! \div 2! = 6 \times 5 \times 5! \div 2! = 1800$$

**Solution 4**

There are 5 subjects and 6 periods. Each subject must be allowed in at least one period. Therefore, two periods will have same subject and remaining four periods will have different subjects.

Select the two periods where the same subject is taught. This can be done in $6C_2$ ways.
Allocate a subject two these two periods ($^5C_1$ ways).

Remaining 4 subjects can be arranged in the remaining 4 periods in 4! ways.

Required number of ways

\[ = ^6C_2 \times ^5C_1 \times 4! = 1800 \]

23. How many 6 digit telephone numbers can be formed if each number starts with 35 and no digit appears more than once?

A. 720  
B. 360  
C. 1420  
D. 1680

Answer: Option D

Explanation:

The first two places can only be filled by 3 and 5 respectively and there is only 1 way for doing this.

Given that no digit appears more than once. Hence we have 8 digits remaining (0,1,2,4,6,7,8,9)

So, the next 4 places can be filled with the remaining 8 digits in $^8P_4$ ways.

Total number of ways = $^8P_4 = 8 \times 7 \times 6 \times 5 = 1680$

24. An event manager has ten patterns of chairs and eight patterns of tables. In how many ways can he make a pair of table and chair?

A. 100  
B. 80  
C. 110  
D. 64

Answer: Option C
Answer: Option B

Explanation:

He has 10 patterns of chairs and 8 patterns of tables

A chair can be selected in 10 ways.
A table can be selected in 8 ways.

Hence one chair and one table can be selected in \(10 \times 8 = 80\) ways.

25. 25 buses are running between two places P and Q. In how many ways can a person go from P to Q and return by a different bus?

A. None of these  
B. 600  
C. 576  
D. 625

Answer: Option B

Explanation:

He can go in any of the 25 buses (25 ways).
Since he cannot come back in the same bus, he can return in 24 ways.

Total number of ways = \(25 \times 24 = 600\)

26. A box contains 4 red, 3 white and 2 blue balls. Three balls are drawn at random. Find out the number of ways of selecting the balls of different colours?

A. 62  
B. 48  
C. 12  
D. 24

Answer: Option B

Explanation:

He can go in any of the 25 buses (25 ways).
Since he cannot come back in the same bus, he can return in 24 ways.

Total number of ways = \(25 \times 24 = 600\)
Answer: Option D

Explanation:

1 red ball can be selected in $4C_1$ ways. 
1 white ball can be selected in $3C_1$ ways. 
1 blue ball can be selected in $2C_1$ ways.

Total number of ways 
= $4C_1 \times 3C_1 \times 2C_1$ 
= $4 \times 3 \times 2 = 24$

27. A question paper has two parts P and Q, each containing 10 questions. If a student needs to choose 8 from part P and 4 from part Q, in how many ways can he do that?
A. None of these       B. 6020 
C. 1200           D. 9450

**Answer with explanation**

**Answer: Option D**

**Explanation:**

Number of ways to choose 8 questions from part P = $10C_8$ 
Number of ways to choose 4 questions from part Q = $10C_4$

Total number of ways 
= $10C_8 \times 10C_4$ 
= $10C_2 \times 10C_4$ [∵ $nC_r = nC_{n-r}$] 
= $(10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) \times (10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1)$ 
= $45 \times 210 = 9450$

28. In how many different ways can 5 girls and 5 boys form a circle such that the boys and the girls alternate?
A. 2880       B. 1400 
C. 1200       D. 3212
Answer with explanation

Answer: Option A

Explanation:

Around a circle, 5 boys can be arranged in 4! ways.

Given that the boys and the girls alternate. Hence there are 5 places for the girls. Therefore the girls can be arranged in 5! ways.

Total number of ways 
=4!×5!=24×120=2880=4!×5!=24×120=2880

29. Find out the number of ways in which 6 rings of different types can be worn in 3 fingers?

A. 120  
B. 720  
C. 125  
D. 729

Answer with explanation

Answer: Option D

Explanation:

The first ring can be worn in any of the 3 fingers (3 ways).

Similarly each of the remaining 5 rings also can be worn in 3 ways.

Hence total number of ways 
=3×3×3×3×3×3=36=729=3×3×3×3×3×3=36=729

30. In how many ways can 5 man draw water from 5 taps if no tap can be used more than once?

A. None of these  
B. 720  
C. 60  
D. 120
Answer with explanation

Answer: Option D

Explanation:

1\textsuperscript{st} man can draw water from any of the 5 taps.
2\textsuperscript{nd} man can draw water from any of the remaining 4 taps.
3\textsuperscript{rd} man can draw water from any of the remaining 3 taps.
4\textsuperscript{th} man can draw water from any of the remaining 2 taps.
5\textsuperscript{th} man can draw water from remaining 1 tap.

\[
\begin{array}{c|c|c|c|c|}
5 & 4 & 3 & 2 & 1 \\
\end{array}
\]

Hence total number of ways

\[5 \times 4 \times 3 \times 2 \times 1 = 120\]