**SIMPLE & COMPOUND INTEREST**

**INTEREST**
It is money paid by borrower for using the lender's money for a specified period of time.
Denoted by $I$.

**PRINCIPAL**
The original sum borrowed. Denoted by $P$.

**TIME**
Time period for which the money is borrowed. Denoted by $n$.

**RATE OF INTEREST**
Rate at which interest is calculated on the original sum. Denoted by $r$.

**AMOUNT**
Sum of Principal plus Interest. Denoted by $A$.

**SIMPLE INTEREST**
The interest calculated every year on original principal, i.e. the sum at the beginning of first year. Denoted by $SI$.

\[ SI = Pnr \]

\[ A = P + SI \]

**COMPOUND INTEREST**
The interest is added to the principal at the end of each period to arrive at the new principal for the next period.

**OR**

The amount at the end of year will become principal for the next year and so on.

Let $P$ be principal borrowed at the beginning of period 1.

Amount at end of period $n=1$ is
A= \( P \left(1 + \frac{r}{100}\right) \)

Then,

New Principal at the beginning of period 2 will be A i.e. \( P \left(1 + \frac{r}{100}\right) = P \times R \) where \( R = (1 + \frac{r}{100}) \).

**Let's checkout the applicability of the above concept with an example**

Consider P at the beginning of year of Rs 100 and \( r=10\% \) p.a. Now, for the next three years the calculation of simple and compound interest is as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Under Simple Interest</th>
<th>Under compound interest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Principal at beginning of year</td>
<td>Interest for the year</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>10</td>
</tr>
</tbody>
</table>

As can be seen from table, 

<table>
<thead>
<tr>
<th>UNDER SIMPLE INTEREST</th>
<th>UNDER COMPOUND INTEREST</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P ) is same for every year</td>
<td>( A ) at the end of every year = ( P ) for next year</td>
</tr>
<tr>
<td>( I ) is same for every year</td>
<td>( I ) is different for each year.</td>
</tr>
</tbody>
</table>

**IMPORTANT FORMULAE**

Let Principal = Rs. \( P \), Time = \( t \) yrs and Rate = \( r \% \) per annum
Q 1.

In what time will Rs 390625 amount to Rs 456976 at 4% compound interest?

Solution: 

\[ P \left(1 + \frac{r}{100}\right)^t = A \]

\[ 390625 \left(1 + \frac{4}{100}\right)^t = 456976 \]

\[ \left(1 + \frac{1}{25}\right)^t \times \frac{456976}{390625} = \frac{456976}{390625} \]

Therefore, 

\[ \left[ \frac{26}{25} \right]^t = \left[ \frac{26}{25} \right]^4 \]

Therefore, \( t = 4 \) the required time is 4 years.
Q2.

A sum of money placed at compound interest doubles itself in 4 yrs. In how many years will it amount to eight times itself?

Solution :-

Quicker Approach:

X becomes 2x in 4 yrs.

2x becomes 4x in next 4 yrs.

4x becomes 8x in yet another 4 yrs.

Thus, x becomes 8x in 4 + 4 + 4 = 12 yrs.

Q 3.

Find the least number of complete years in which a sum of money at 20% CI will be more than doubled.

Solution: We have, \( P \left[ 1 + \frac{20}{100} \right]^t > 2P \)

By trial, \( \frac{6}{5} \times \frac{6}{5} \times \frac{6}{5} \times \frac{6}{5} > 2 \)

\( \therefore \left[ \frac{6}{5} \right]^t > 2 \)

Therefore the required time is 4 yrs.

Q 4.

A sum of money at compound interest amounts to thrice itself in three years. In how many years will it be 9 times itself?
Fast Method: Remember the following conclusion:

If a sum becomes $x$ times in $y$ years at CI then it will be $(x)^n$ times in $ny$ years.

Thus, if a sum becomes 3 times in 3 years it will be $(3)^2$ times in $2 \times 3 = 6$ years.

Example: If a sum deposited at compound interest becomes double in 4 years when will it be 4 times at the same rate of interest?

Solution: Using the above conclusion, we say that the sum will be $(2)^2$ times in $2 \times 4 = 8$ years.

TO FIND RATE

Q 1.

At what rate per cent compound interest does a sum of money become nine-fold in 2 years?

Solution :-
At what rate percentage (compound interest) will a sum of money become eight times in three years?

Solution: By direct formula:

\[
\text{Rate} \% \left[ (8)^{\frac{1}{3}} - 1 \right] \times 100.
\]

\[
= \left[ (8)^{\frac{1}{3}} - 1 \right] \times 100 = (2 - 1) \times 100 = 100\%
\]

At what rate per cent compounded yearly will be Rs. 80,000 amount to Rs 88,200 in 2 years?

Solution: We have,

\[
80,000 \left[ 1 + \frac{r}{100} \right]^2 = 88,200
\]

Or,

\[
\left[ 1 + \frac{r}{100} \right]^2 = \frac{88,200}{80,000} = \frac{441}{400} = \left[ \frac{21}{20} \right]^2
\]

Or,

\[
1 + \frac{r}{100} = \frac{21}{20} \quad \therefore \quad r = 5\%
\]
GIVEN CI, To find SI and vice versa

**Q1.**

*If the CI on a certain sum for 2 years at 3% be Rs. 101.50, what would be the SI?*

**Solution:**

\[
\text{Cl on 1 rupee} = \left(1 + \frac{3}{100}\right)^2 - 1 = \left(\frac{103}{100}\right)^2 - 1 = \text{Re} \left(\frac{609}{10000}\right)
\]

\[
\therefore \text{SI on 1 rupee} = \text{Re} \left(\frac{2 \times 3}{100}\right) = \text{Re} \left(\frac{6}{100}\right)
\]

\[
\therefore \left[\frac{\text{SI}}{\text{Cl}}\right] = \left[\frac{6}{100}\right] \times \left[\frac{609}{10000}\right] = \frac{200}{203}
\]

\[
\therefore \text{SI} = \frac{200}{203} \text{ of Cl} = \frac{200}{203} \times 101.5 = \text{Rs. 100}
\]

GIVEN CI AND SI, TO FIND SUM AND RATE

**Q1.**

*The compound interest on a certain sum for 2 yrs is Rs 40.80 and simple interest is Rs. 40.00. Find the rate of interest per annum and the sum.*

**Solution:** A little reflection will show that the difference between the simple and compound interests for 2 yrs is the interest on the first year’s interest.

First year’s SI = Rs 40/2 = Rs 20

CI – SI = Rs 40.8 – Rs 40 = Rs 0.80

Interest on Rs 20 for 1 year = Re 0.80
Interest on Rs 100 for 1 yr = Rs \frac{80 \times 100}{100 \times 20} = Rs 4

\therefore \text{Rate} = 4\%

Now, principal \( P \) is given by

\[ P = \frac{\text{100} \times \text{1}}{\text{tr}} = \frac{\text{100} \times \text{40}}{\text{2} \times \text{4}} = \text{Rs. 500} \]

Quicker Method (Direct formula) [for 2 yrs only]

\[ \text{Rate} = \frac{2 \times \text{Difference in CI and SI}}{\text{SI}} \times 100 \]

Thus, in this case, rate \( = \frac{2 \times 0.8}{40} \times 100 = 4\% \)

And sum \( = \frac{40 \times 100}{4 \times 2} = \text{Rs. 500} \)

**MODERATE BASE PRACTICE QUE. SET**

Q1. Mr. Kuldeep invested an amount of Rs. 13,900 divided in two different schemes A and B at the simple interest rate of 14% p.a. and 11% p.a. respectively. If the total amount of simple interest earned in 2 years be Rs. 3508, what was the amount invested in Scheme B?

a) Rs. 6400  
b) Rs. 6500  
c) Rs. 7200  
d) Rs. 7500  
e) None of these

Q2. How much time will it take for an amount of Rs. 450 to yield Rs. 81 as interest at 4.5% per annum of simple interest?

a) 3.5 years  
b) 4 years  
c) 4.5 years  
d) 5 years  
e) None of these
Q3. A sum of Rs. 12,500 amounts to Rs. 15,500 in 4 years at the rate of simple interest. What is the rate of interest?

a) 3%  
b) 4%  
c) 5%

d) 6%  
e) None of these

Q4. An automobile financier claims to be lending money at simple interest, but he includes the interest every six months for calculating the principal. If he is charging an interest of 10%, the effective rate of interest becomes:

a) 10%  
b) 10.25%  
c) 10.5%

d) Data inadequate  
e) None of these

Q5. Natasa lent Rs. 5000 to Gita for 2 years and Rs. 3000 to Sita for 4 years on simple interest at the same rate of interest and received Rs. 2200 in all from both of them as interest. The rate of interest per annum is:

a) 5%  
b) 7%  
c) 7 1/8%

d) 10%  
e) None of these

Q6. Rahul took loan from a bank at the rate of 12% p.a. simple interest. After 3 years he had to pay Rs. 5400 interest only for the period. The principal amount borrowed by him was:

a) Rs. 2000  
b) Rs. 10,000  
c) Rs. 15,000

d) Rs. 20,000  
e) None of these

Q7. What will be the ratio of simple interest earned by certain amount at the same rate of interest for 6 years and that for 9 years?

a) 1 : 3  
b) 1 : 4  
c) 2 : 3
d) Data inadequate    e) None of these

Q8. Sandeep borrows Rs. 5000 for 2 years at 4% p.a. simple interest. He immediately lends it to another person at 6 ¼% pa for 2 years. Find his gain in the transaction per year.
   a) Rs. 112.50   b) Rs. 125   c) Rs. 150
   d) Rs. 150    d) Rs. 167.50

Q9. On a sum of money, the simple interest for 2 years is Rs.660, while the compound interest is Rs.696.30, the rate of interest being the same in both the cases. The rate of interest is:
   a) 10%   b) 10.5%   c) 12%
   d) Data inadequate   e) None of these

Q10. Mr. Ram Singh invested an amount of Rs.13,900 divided in two different schemes A and B at the simple interest rate of 14% p.a. and 11% p.a. respectively. If the total amount of simple interest earned in 2 years be Rs.3508, what was the amount invested in Scheme B?
   a) Rs.6400   b) Rs.6500   c) Rs.7200
   d) Rs.7500    e) None of these

Q11. What should be the least number of years in which the simple interest on Rs.2600 at [6(2/3)]% will be an exact number of rupees?
   a) 2   b) 3   c) 4
   d) 5    e) None of these

Q12. An amount of Rs.1,00,000 is invested in two types of shares. The first yields an interest of 9% p.a. and the second, 11% p.a. If the total
interest at the end of one year is \(\left[9\frac{3}{4}\right]\%\), then the amount invested in each share was:

a) Rs.52,500, Rs.47,500  
b) Rs.62,500, Rs.37,500  
c) Rs.72,500, Rs.27,500  
d) Rs.82,500, Rs.17,500  
e) None of these

Q13. If the simple interest on a certain sum for 15 months at \([7 \left(\frac{1}{2}\right)]\%\) per annum exceeds the simple interest on the same sum for 8 months at \([12 \left(\frac{1}{2}\right)]\%\) per annum by Rs.32.50, then the sum (in Rs.) is:

a) Rs.3000  
b) Rs.3060  
c) Rs.3120  
d) Rs.3250  
e) None of these

Q14. A sum of money trebles itself in 15 years 6 months. In how many years would it double itself?

a) 6 years 3 months  
b) 7 years 9 months  
c) 8 years 3 months  
d) 9 years 6 months  
e) None of these

Q15. Sarita took a loan of Rs. 1200 with simple interest for as many years as the rate of interest. If he paid Rs. 432 as interest at the end of the loan period, what was the rate of interest?

a) 3.6  
b) 6  
c) 18  
d) Data inadequate  
e) None of these

ANSWERS

1. Option A

Let the sum invested in scheme A be Rs. x and that in scheme B be Rs. \(13900 - x\)
Then, \[ \left[ x \times 14 \times 2 / 100 \right] \div \left[ \{(13,900 - x) \times 11 \times 2 \} / 100 \right] = 3508 \]

\[ 28x - 22x = 350800 - (13900 \times \times 2) \]
\[ 6x = 45000 \]
\[ x = 7500 \]
So, sum invested in Scheme B = Rs. (13900 - 7500) = Rs.6400

2. **Option B**

Time = \[ 100 \times 81 / 450 \times 4.5 \] years = 4 years

3. **Option D**

S.I. = Rs. (15500 - 12500) = Rs.3000
Rate = \[ 100 \times 3000 / 12500 \times 4 \] % = 6%

4. **Option B**

Let the sum be Rs.100. Then,
S.I. for first 6 months = Rs. \[ 100 \times 10 \times 1 / 100 \times 2 \] = Rs.5
S.I. for last 6 months = Rs. \[ 105 \times 10 \times 1 / 100 \times 2 \] = Rs.5.25
So, amount at the end of 1 year = Rs. (100 + 5 + 5.25) = Rs.110.25
So, effective rate = (110.25 - 100) = 10.25%

5. **Option D**

Let the rate be R% p.a.
Then, \[ 500 \times R \times 2 / 100 \] + \[ 300 \times R \times 4 / 100 \] = 2200
\[ 100R + 120R = 2200 \]
\[ R = \left[ 2200 / 220 \right] = 10 \]
So, rate = 10%

6. **Option C**
Principal = Rs. \([100 \times 5400 / 12 \times 3]\) = Rs.15000

7. **Option C**

   Let the principal be \(P\) and rate of interest be \(R\%\).
   So, required ratio = \([P \times R \times 6 / 100] / [P \times R \times 9 / 100]\) = \(6PR / 9PR = 6 / 9 = 2 : 3\)

8. **Option A**

   Gain in 2 years = Rs. \([5000 \times 25 / 4 \times 2 / 100] - (5000 \times 4 \times 2 / 100)\]
   = Rs. \((625 - 400)\)
   = Rs.225
   So, gain in 1 year = Rs. \(225 / 2\) = Rs.112.50

9. **Option E**

   Difference in C.I. and S.I. for 2 years – Rs. \((696.30 - 660)\) = Rs.36.30
   S.I. for one year = Rs.330

10. **Option A**

    Let the sum invested in Scheme A be Rs. \(x\) and that in Scheme B be Rs. \((13900 - x)\)
    Then, \(x \times 14 \times 2 / 100 + [(13,900 - x) \times 11 \times 2 / 100]\) = Rs.3508
    \(28x - 22x = 350800 - (13900 \times 22)\)
    \(6x = 45000\)
    \(x = 7500\)
    So, sum invested in Scheme B = Rs. \((13900 - 7500)\) = Rs.6400

11. **Option B**

    S.I. = Rs. \([2600 \times 20 / 3 \times 1 / 100 \times T]\) = Rs. \([520 / 3 \times T]\)
12. Option B

Let the sum invested at 9% be Rs. x and that invested at 11% be Rs. \(100000 - x\)

Then, \[
\frac{x \times 9 \times 1}{100} + \frac{(100000 - x) \times 11 \times 1}{100} = \frac{100000 \times 39}{4 \times 1 \times 100}
\]

\[
9x + \frac{1100000 - 11x}{100} = \frac{390000}{4} = 9750
\]

\[
2x = (1100000 - 975000) = 125000
\]

\[
x = 62500
\]

Sum invested at 9% = Rs.62500

Sum invested at 11% = Rs. (100000 - 62500) = Rs.37500

13. Option C

Let the sum be Rs. x. Then, \[
\frac{x \times 15}{2} \times \frac{5}{4} \times \frac{1}{100} - \frac{x \times 25}{2} \times \frac{2}{3} \times \frac{1}{100} = 32.50
\]

\[
75x / 8 - 25x / 3 = 3250
\]

\[
25x = (3250 \times 24)
\]

\[
x = \frac{3250 \times 24}{25} = 3120
\]

14. Option B

Let sum = x. Then, S.I. = 2x, Time = 15 (1/2) years = 31 / 2 years

So, rate = \[
\frac{100 \times 2x / x \times (31/2)}{x} = 400 / 31%
\]

Now, sum = x, S.I. = x, Rate = 400 / 31%

So, time = \[
100 \times x / x \times (400/31) = 31 / 4 \] years = 7 years 9 months

15. Option B

Let rate = R% and time = R years

Then, \[
\frac{1200 \times R \times R}{100} = 432
\]

\[
12 R^2 = 432
\]

\[
R^2 = 36
\]

\[
R = 6
\]